Math 132 Final Exam Fall 2018

- 20 multiple choice questions worth 5 points each.
- NO hand graded questions.
- Exam covers everything!
  4.4, 4.9, 5.1-5.5, 6.1-6.3, 6.5, 7.1-7.5, 7.8 8.1-8.2, 11.1-11.11.

- No calculators!
- For the multiple choice questions, mark your answer on the answer card.

Useful Formulas

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$
$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$
$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$	$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$
$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$	$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$\int \csc x \, dx = -\ln\lvert\csc x + \cot x\rvert + C$	$\int \sec x \, dx = \ln\lvert\sec x + \tan x\rvert + C$
$\cosh t = \frac{1}{2}(e^t + e^{-t})$	$\sinh t = \frac{1}{2}(e^t - e^{-t})$
$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$
$\cosh^2 t = 1 + \sinh^2 t$	

1. Approximate the area under the curve $y = x^3 - 1$ on the interval $[-2, 4]$ using a right-hand Riemann sum with three subdivisions.

   A. -6

   B. 48

   C. 54

   D. 66

   E. 92

   F. 138

2. What is the integral corresponding to the Riemann sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2 + \frac{5i}{n} \right) + \ln \left(2 + \frac{5i}{n} \right) \right] \cdot \frac{5}{n}$$

A. $\int_{2}^{5} (x + \ln(x))\, dx$

B. $\int_{0}^{5} 5\,(x + \ln(x))\, dx$

C. $\int_{2}^{7} (x + \ln(x))\, dx$

D. $\int_{2}^{7} (x + 2 + \ln((x + 2)))\, dx$

3. Compute

$$\frac{d}{dx}\int_0^{x^3} e^{t^2}\ dt$$

A. e^{x^2}
B. $2x^3e^{x^6}$
C. $\dfrac{e^{x^6}}{2x^3}$
D. $3x^2e^{x^6}$
E. e^{x^6}

4. Evaluate

$$\int \frac{y}{e^{5y}}\, dy$$

A. $-\frac{1}{5}e^{-5y} + C$

B. $-\frac{1}{5}e^{-5y}(5y+1) + C$

C. $-\frac{1}{25}e^{-5y}(y+5) + C$

D. $-\frac{1}{25}e^{-5y}(y+1) + C$

E. $-\frac{1}{25}e^{-5y}(5y+1) + C$

5. Evaluate

$$\int \frac{dx}{x^2 + 64}$$

A. $\ln(x^2 + 64) + C$

B. $\frac{1}{2}\ln(x^2 + 64) + C$

C. $\arctan(x) + C$

D. $\arctan\left(\frac{x}{8}\right) + C$

E. $\frac{1}{8}\arctan\left(\frac{x}{8}\right) + C$

F. $\arctan\left(\frac{x}{64}\right) + C$

G. $\frac{1}{64}\arctan\left(\frac{x}{64}\right) + C$

6. Find

$$\int \frac{dx}{x^2 - 64}$$

A. $\frac{1}{2x} \ln \left| \frac{x-8}{x+8} \right| + C$

B. $\frac{1}{16} \ln \left| \frac{x-8}{x+8} \right| + C$

C. $\frac{1}{16} \ln \left| \frac{x+8}{x-8} \right| + C$

D. $\frac{1}{2x} \ln \left| \frac{x+8}{x-8} \right| + C$

7. Find

$$\int_0^{\pi/4} \cos(3x)\cos(x)\,dx$$

A. -1

B. $-\frac{1}{2}$

C. $-\frac{1}{4}$

D. 0

E. $\frac{1}{4}$

F. $\frac{1}{2}$

G. 1

8. Evaluate

$$\int \sin^3 x \cos^7 x \, dx$$

A. $\dfrac{\sin^4 x + \cos^8 x}{2} + C$

B. $-\dfrac{\cos^8 x}{8} + \dfrac{\cos^{10} x}{10} + C$

C. $\dfrac{\cos(8x)}{8} - \dfrac{\cos(10x)}{10} + C$

D. $\dfrac{\cos^8 x}{8} - \dfrac{\cos^{10} x}{10} + C$

E. $\dfrac{\cos^8 x}{24} + C$

9. Find the area between the curves x^2 and x^3.

A. 1

B. $\frac{1}{12}$

C. $\frac{5}{9}$

D. π

E. e

F. $\frac{4}{5}$

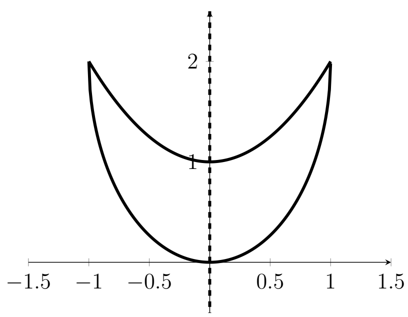
10. A cup of coffee is being stirred, and because of the centrifugal force of the spinning liquid, the coffee surface has vertical section of parabolic shape. The coffee surface can be described as rotating curve below about the y-axis:

$$y = 1 + x^2, \qquad -1 \le x \le 1$$

The shape of cup is the curve below rotated about the y-axis:

$$y = 2 - \sqrt{4 - 4x^2} \qquad -1 \le x \le 1$$

Set up the integral to calculate the volume of the coffee in the cup by **cylindrical shell method**.



A. $\displaystyle\int_0^1 \pi\left[(1+x^2)^2 - (2-\sqrt{4-4x^2})^2\right]dx$

B. $\displaystyle\int_0^1 2\pi x\left[(1+x^2)^2 - (2-\sqrt{4-4x^2})^2\right]dx$

C. $\displaystyle\int_0^1 2\pi x\left[1+x^2-2+\sqrt{4-4x^2}\right]dx$

D. $\displaystyle\int_0^2 2\pi y\left[\sqrt{y-1} - \frac{1}{2}\sqrt{4y-y^2}\right]dy$

E. $\displaystyle\int_0^2 \pi\left[\sqrt{y-1} - \frac{1}{2}\sqrt{4y-y^2}\right]dy$

F. $\displaystyle\int_0^2 \pi\left[(y-1) - \frac{1}{4}(4y-y^2)\right]dy$

11. Let R be the region in quadrant I bounded by the curves $y = x$ and $y = x^3$. Calculate the volume of the revolution solid obtained by rotating R about the x-axis.

    A. 0

    B. $\frac{4\pi}{21}$

    C. $\frac{5\pi}{21}$

    D. $\frac{4}{15}$

    E. $\frac{\pi}{3}$

    F. $\frac{8\pi}{21}$

    G. $\frac{7\pi}{15}$

12. The arc length of what function, $f(x)$, is being calculated by the following integral:

$$\int_0^2 \sqrt{1 + \frac{\sin^2(\sqrt{x}))}{4x}}\, dx$$

A. $f(x) = \dfrac{\sin^2(\sqrt{x}))}{4x}$

B. $f(x) = \cos(\sqrt{x})$

C. $f(x) = \dfrac{\sin(\sqrt{x}))}{x}$

D. $f(x) = \sin^2(\sqrt{x})$

E. $f(x) = \cos^2(\sqrt{x})$

F. $f(x) = e^x$

13. Rotate the curve $y = \dfrac{1}{x^2}$, $1 \leq x \leq 4$ about the y-axis. Find the integral that gives the surface area.

A. $\displaystyle\int_1^4 2\pi \frac{1}{x^2}\sqrt{1 + \frac{4}{x^6}}\, dx$

B. $\displaystyle\int_1^4 2\pi \sqrt{1 + \frac{4}{x^6}}\, dx$

C. $\displaystyle\int_1^4 2\pi x \sqrt{1 + \frac{4}{x^6}}\, dx$

D. $\displaystyle\int_1^4 2\pi x \sqrt{1 + \frac{1}{x^4}}\, dx$

E. $\displaystyle\int_{1/4}^1 2\pi y \, dy$

14. Take the limit of the sequence:

$$\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$$

A. Diverges to ∞

B. Converges to e

C. Converges to 1

D. Converges to π

E. Converges to $\pi^2/6$

F. Converges to 0

15. Exactly how many of the following series converge?

(I) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2 - n^3}{2 - 8n^3}$

(II) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(III) $\displaystyle\sum_{n=2}^{\infty} \frac{3^n}{n \cdot (-2)^{n+1}}$

(IV) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

(V) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\cos(n)}{n}\right)^n$

A. 1

B. 2

C. 3

D. 4

E. 5

16. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}$$

A. $(-1, 1)$

B. $(2, 4)$

C. $(2, 4]$

D. $[2, 4)$

E. $[2, 4]$

17. Which of the following has the Taylor series:

$$\ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^n x^n}{n 2^n} = \ln 2 + \left(\frac{3}{2}\right) x - \frac{1}{2}\left(\frac{3}{2}\right)^2 x^2 + \frac{1}{3}\left(\frac{3}{2}\right)^3 x^3 - \frac{1}{4}\left(\frac{3}{2}\right)^4 x^4 + \cdots$$

A. $\frac{1}{(2+3x)^2}$, centered at $x = 0$

B. $\ln(3 + 2x)$, centered at $x = 0$

C. $\ln(3 + 2x)$, centered at $x = -3$

D. $\ln(2 + 3x)$, centered at $x = 0$

E. $\ln(2 + 3x)$, centered at $x = -2$

18. Which of the following is a power series representation for $\dfrac{1}{x+2}$?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n+1}$

B. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n}$

C. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n}} x^{n}$

D. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n}$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^{n+1}$

F. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n}} x^{n}$

19. Find a power series for:

$$f(x) = x^3 \arctan(x^4)$$

Use this to find the 15th derivative of $f(x)$ evaluated at $x = 0$.

$$f^{(15)}(0) = ?$$

A. $-15!$
B. $-\dfrac{15!}{3}$
C. -5
D. -1
E. 0
F. $\dfrac{1}{3}$
G. 1
H. 5
I. $\dfrac{15!}{3}$
J. $15!$

20. Find the value of the series, or conclude that it diverges.

$$\sum_{n=0}^{\infty} \frac{(-1)^n(\pi/3)^{2n}}{(2n)!} = 1 - \frac{(\pi/3)^2}{2!} + \frac{(\pi/3)^4}{4!} - \frac{(\pi/3)^6}{6!} + \frac{(\pi/3)^8}{8!} - \frac{(\pi/3)^{10}}{10!} + \cdots$$

A. Diverges

B. $-\frac{\pi^2}{2}$

C. -1

D. $-\frac{1}{2}$

E. $-\frac{1}{3}$

F. 0

G. $\frac{1}{3}$

H. $\frac{1}{2}$

I. 1

J. $\frac{\pi^2}{2}$