Math 132 Course Review

1. Integral Basics $\boxed{\textbf{3 Questions}}$

   (a) Left and right hand sums. $\boxed{\text{Find a left/right hand with 5 subdivisions.}}$

   (b) Riemann Sums. $\boxed{\text{Which integral is represented by the given Riemann sum?}}$
   $\boxed{\text{Find a limit of Riemann sums for the given integral.}}$

   (c) FTC, parts 1 and 2. $\boxed{\text{Find the derivative of an integral function.}}$
   $\boxed{\text{Find a definite integral.}}$

2. Integration Techniques $\boxed{\textbf{5 Questions}}$ $\boxed{\text{Compute an integral using the techniques below.}}$

   (a) Subsitution
   (b) Parts
   (c) Trig
   (d) Trig Sub
   (e) Partial Fractions

   Q: How do you recognize when to use each of these techniques?

3. Integration Applications $\boxed{\textbf{5 Questions}}$

   (a) Areas $\boxed{\text{Compute a given area.}}$

   (b) Improper integrals (Type I and II) $\boxed{\text{Compute an improper integral.}}$

   (c) Volumes: Washers, Shells $\boxed{\text{Set up and compute volume.}}$

   (d) Average Value $\boxed{\text{Find average value.}}$

   (e) Arc Length $\boxed{\text{Find arc length.}}$

   (f) Surface Area $\boxed{\text{Find surface area.}}$

4. Series $\boxed{\textbf{7 Questions}}$

   (a) Sequences and Series, partial sums. $\boxed{\text{Find the limit of a given sequence.}}$

   (b) Geometric Series $\boxed{\text{Find the sum of a geometric series.}}$

   (c) Telescoping Series $\boxed{\text{Find the sum of a telescoping series.}}$

   (d) Convergence tests: Integral, Comparison, Limit Comparison, Alternating Series Test, Ratio Test, Root Test
   $\boxed{\text{Given a series, does it converge or diverge?}}$
   $\boxed{\text{Maximum error (remainder) for an alternating series?}}$

   (e) Power Series and Taylor Series. $\boxed{\text{Radius and Interval of Convergence?}}$
   $\boxed{\text{Given a function, find its power series.}}$
   $\boxed{\text{Given a power series, find its function.}}$
   $\boxed{\text{Given a series, can you recognize it as a power series evaluated at a given point?}}$
   $\boxed{\text{Can you use the table of derivatives to find a power series?}}$

   (f) Frequently used power series: $\dfrac{1}{1-x}$, $\ln(1+x)$, $\arctan x$, e^x, $\sin x$, $\cos x$.

Highlighted Questions

1. Review #52 Approximate $\int_0^3 x^2 \, dx$ by using the midpoint rule and 3 subdivisions.   **Answer:** $35/4$

2. Review #53 Let $g(x) = \int_2^{1/x} \arctan t \, dt$. Find $g'(x)$.   **Answer:** $(-1/x^2)\arctan(1/x)$

3. Review #2 Find $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}} \, dx$ **Answer:** Substitution: 4

4. Review #7 Find $\displaystyle\int_0^1 x \sin(2x) \, dx$.   **Answer:** Parts: $\frac{1}{4}\sin 2 - \frac{1}{2}\cos 2$

5. Review #12 Find $\int_0^\infty \frac{x}{(x^2+2)^2} \, dx$ **Answer:** Improper: $1/4$

6. Review #8 Find the length of $y = (2/3)x^{3/2}$, $0 \le x \le 3$.   **Answer:** Arc length: $14/3$

7. Review #56 $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$.
   How many terms must be added to obtain a partial sum within $\frac{1}{100}$ of $\ln 2$?   **Answer:** 99 terms.

8. Review #1-e Converge or Diverge: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^\pi}$ **Answer:** p-series with $p = \pi$. Converges.

9. Review #15 $x \cos(x/2) = \displaystyle\sum_{n=0}^{\infty} a_n x^n$. Find a_5.   **Answer:** $1/384$

10. Review #16 $\displaystyle\int \arctan(x^2) \, dx = C + \sum_{n=1}^{\infty} a_n x^n$, find a_7.   **Answer:** $-1/21$

11. Review #17 $\sin x = \displaystyle\sum_{n=0}^{\infty} a_n (x - \pi/3)^n$. Find a_3.   **Answer:** $-1/12$

12. Review #27 Find a function represented by: $1 + 3x + 9x^2 + 27x^3 + 81x^4 + \cdots$ **Answer:** $1/(1 - 3x)$

13. Review #38 Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n (x+3)^n}{4^n \sqrt{n+1}}$ **Answer:** $R = 4$. $I = (-7, 1]$

14. Review #37 Let $f(x) = \frac{1}{72} x^3 e^{2x^2}$. Find $f^{(7)}(0)$.   **Answer:** Use power series: 140.

15. Find a series for $\sinh x = \dfrac{1}{2}(e^x - e^{-x})$.   **Answer:** $\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

16. Find a series for 2^x, centered at $x = 1$.   **Answer:** $2^x = \sum_{n=0}^{\infty} \frac{2(\ln 2)^n (x-1)^n}{n!}$

17. Find a series for $x \cos(x^3)$ and use it to approximate $\displaystyle\int_0^1 x \cos(x^3) \, dx$. How accurate is the 5th partial sum?

   Answer: 5th partial sum: $1 - \frac{1}{8\cdot 2!} - \frac{1}{14\cdot 4!} - \frac{1}{20\cdot 6!} - \frac{1}{26\cdot 8!}$. Next term: $\frac{1}{32\cdot 10!} = 8.6 \times 10^{-9}$.