Math 132 - November 27, 2017

Section 11.7: Series Strategy

Section 11.8: Power Series

- Goal: Apply what we've learned to series like these:

$$\sum_{n=0}^{\infty} a_n x^n \qquad \text{and} \qquad \sum_{n=0}^{\infty} a_n (x-a)^n$$

- For what values of x does a power series converge?
  - Radius of Convergence
  - Interval of Convergence
- For $\sum a_n(x-a)^n$, three possibilities.
  - (i) The series converges for only $x = a$.
  - (ii) The series converges for all x.
  - (iii) There is a positive number R and series converges for $|x-a| < R$ but diverges for $|x-a| > R$.

1. **Clicker** Select the easiest x value to plug in to the power series $\sum_{n=0}^{\infty} x^n$

   (a) $x = 0$ (b) $x = 1$ (c) $x = -1$ (d) No value is easy to plug in.

2. **Clicker** Select the easiest x value to plug in to the power series $\sum_{n=0}^{\infty} \frac{(x+1)^n}{n^2+1}$

   (a) $x = 0$ (b) $x = 1$ (c) $x = -1$ (d) No value is easy to plug in.

3. Determine the Radius of Convergence and Interval of Convergence for the power series below.

   $\sum_{n=0}^{\infty} x^n$

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4. What is the difference between a **sequence** and a **series**?
5. What does it mean for a sequence to **diverge**? **Converge?**
6. What does it mean for a series to **diverge**? **Converge?**
7. What is a **partial sum** and what are they good for?
8. Give an example of a convergent **geometric series**.
9. Give an example of a divergent **geometric series**.
10. What is the **Test for Divergence** and what is it good for?
11. What is the **Integral Test** and what is it good for?
12. What is a **p Series**?
13. What is the **Comparison Test**?
14. What is the **Limit Comparison Test**?
15. What is an **Alternating Series**?
16. What is the **Alternating Series Test**?
17. What is **Conditionally Convergent**?
18. What is the **Absolutely Convergent**?
19. What is the **Ratio Test**?
20. What is the **Root Test**?
21. **Clicker** Which converge:

    I. $\sum_{n=2}^{\infty} \frac{n-1}{3n+1}$ II. $\sum_{n=1}^{\infty} ne^{-n^2}$

    (a) Neither of them. (b) I only (c) II only (d) I and II

22. **Clicker** Which converge?

    I. $\sum_{n=1}^{\infty} \left(2^{\frac{1}{n}} - 1\right)^n$ II. $\sum_{n=3}^{\infty} \frac{e^{\frac{1}{n}}}{n^2-3}$

    (a) Neither of them. (b) I only (c) II only (d) I and II

23. **Clicker** Which converge?

I. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$ II. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

(a) Neither of them. (b) I only (c) II only (d) I and II

24. **Clicker** Which converge conditionally, but not absolutely?

I. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n n!}{n^n}$ II. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^3}$

(a) Neither of them. (b) I only (c) II only (d) I and II

25. **Clicker** Which converge?

I. $a_n = \dfrac{n^3 + \sqrt{3n^6 + 2n - 1}}{4n^3 + 2n + 5}$ II. $b_n = \dfrac{\cos\left(\frac{1}{n}\right)}{n+1}$

(a) Neither of them. (b) I only (c) II only (d) I and II

26. **Clicker** For which of the following series is the Ratio Test inconclusive?

I. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{2^n}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$

(a) Neither of them. (b) I only (c) II only (d) I and II

27. **Clicker** Which converge?

I. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{3n}}{n!}$

(a) Neither of them. (b) I only (c) II only (d) I and II

28. **Clicker** Which converge?

I. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n + n}{e^{3n} - 3}$

(a) Neither of them. (b) I only (c) II only (d) I and II

29. Name the strategy(s) to use to determine convergence or divergence:

(a) $\displaystyle\sum (0.98)^n$

(b) $\displaystyle\sum (-5)^{-n}$

(c) $\displaystyle\sum \frac{1}{n(n+6)}$

(d) $\displaystyle\sum \frac{(-1)^n}{37n}$

(e) $\displaystyle\sum ne^{-n^2}$

(f) $\displaystyle\sum \frac{2^n}{n!}$

(g) $\displaystyle\sum \frac{n!}{e^n}$

(h) $\displaystyle\sum \frac{n^n}{3^{3n+1}}$