Math 132 - October 9, 2017 (Exam 2 Review)

Solutions

Exam 2: 5.4, 6.1, 6.2, 6.3, 6.5, 7.1, 7.2

1. Calculate $\int_1^2 \frac{e^x}{1-e^x} dx$

   Solution: $\ln(e-1) - \ln(e^2-1)$

2. Let $u = 2t - 1$ and rewrite the integral in the variable u.

   $$\int_2^3 t\sqrt{2t-1}\,dt$$

   Solution: $\int_3^5 \frac{1}{4}(u^{3/2} + u^{1/2})\,dy$

3. Find $\int \frac{dx}{x\ln x} = \ln(\ln x) + C$

4. Compute $\int \frac{x}{x^2-3}\,dx = \frac{1}{2}\ln(x^2-3) + C$

5. Compute $\int_0^{\pi/3} \sin x\ \cos^4 x\,dx = \frac{31}{160}$

6. Compute $\int_0^{\sqrt{\ln 3}} 3x\,e^{-x^2}\,dx = 1$

7. Compute $\int_0^1 \sqrt{x^2 - x^4}\,dx = \frac{1}{3}$

8. Evaluate the definite integral $\int_0^2 \frac{dx}{\sqrt{2x+5}} = 3 - \sqrt{5}$

9. Find $\int \cot(x)\ln(\sin x)\,dx = \frac{1}{2}(\ln(\sin x))^2 + C$

10. Compute the area under $y = \sqrt{x+1}$ from $x = 0$ to $x = 3$.

    Solution: $\frac{14}{3}$

11. Let R be the region bounded by $x = 1$, $x = 2$ and $y = 3x - 1$.
    Find the volume of the solid obtained by rotating the region R about the x-axis.

    Solution: 13π

12. Find $\int_0^{\pi/2} e^{\sin x}\cos x\,dx = e - 1$

13. Find $\int_1^2 x\sqrt{x-1}\,dx$**Solution:** $\frac{16}{15}$

14. Find the area enclosed by $y = x^2$ and $y = x$.

    Solution: $\frac{1}{6}$

15. Find the area enclosed by $y^2 = x + 6$ and $y = x$.

    Solution: $20 + \frac{5}{6}$

16. Find the area enclosed by $y = \frac{\ln x}{x}$ and $y = \frac{(\ln x)^2}{x}$.

    Solution: $\frac{1}{6}$

17. Find the volume of the solid whose base is the disc centered at the origin with radius one, whose cross sections perpendicular to the x-axis are squares.

    Solution: $\frac{16}{3}$

18. Find the volume of the solid whose base is the region $|x| + |y| \leq 1$ and whose vertical cross sections perpendicular to the y axis are semicircles (with diameter along the base).

    Solution: $\frac{\pi}{3}$

19. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = x^3$, $y = 8$ and $x = 0$.

    Solution: $96\pi/5$

20. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the x-axis.

    Solution: $3\pi/10$

21. Find the volume of the solid obtained by rotating the region bounded by $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$ about the horizontal line $y = 3$.

    Solution: $\pi(6\sqrt{2} - \frac{13}{2})$

22. Find the volume of the solid obtained by rotating the region bounded by $y = 4 - x^2$, $x = 0$, and $x = 1$ about the vertical line $x = 2$.

    Solution: $9\pi + \frac{13\pi}{6}$

23. $\int_0^3 \frac{1}{\sqrt{x}}\, dx = 2\sqrt{3}$

24. Find the average value of $|x^2 - 2|$ on $[0, 2]$.
    Solution: $(2/3)(2\sqrt{2} - 1)$

25. Find $\displaystyle\int_1^{e^b} \frac{\cos(\ln t)}{t}\, dt = \sin b$

26. $\int_0^1 te^{\pi t}\, dt = (\pi e^\pi - e^\pi + 1)/(\pi^2)$

27. Find the average value of $y = x^2$ over $[1, 3]$
    Solution: $\frac{13}{3}$

28. Find the number c for which $\sqrt{c}$ is the average value of $\sqrt{x}$ over the interval $[0, 2]$.
    Solution: $C = 8/9$

29. Find the average value of e^x on $[0, \ln 2]$ **Solution:** $1/\ln 2$

30. Find $\int_1^2 \log_2 x\, dx = 2 - 1/\ln 2$

31. $\int_0^{\pi/2} x \sin x\, dx = 1$

32. $\int_0^1 x^2 e^{-x}\, dx = 2 - \frac{5}{e}$

33. $16 \int_1^e x^3 \ln x\, dx = 3e^4 + 1$

34. $\int_0^{\pi/2} \sin^2 x \cos^3 x\, dx = 2/15$

35. Let R be region above x-axis and below $y = (\sin x)/x$, $0 \leq x \leq \pi/2$. Rotate R about y axis and find volume.
    Solution: 2π

36. Suppose $f(x) = x^2$ and that $f(7)$ is equal to the average value of f on the interval $[2, b]$. What is b?
    Solution: 11

37. $\int_0^{\pi/2} t \cos t\, dt = (\pi - 2)/2$

38. $\int_0^{\pi/4} \sqrt{\sec^2 x - 1}\, dx = \ln \sqrt{2}$

39. $\int \arcsin x\, dx = x \arcsin x + \sqrt{1 - x^2} + C$
    Note: $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

40. Let R be the region in the first quadrant enclosed by $y = x^2 + 2$, $y = 6$ and $x = 0$. Rotate R about the x axis. Using shell method, find the integral representing the volume.
    Solution: $2\pi \int_2^6 y\sqrt{y - 2}\, dy$

41. Suppose we know $f(1) = 0$, $f(2) = 1$, $\int_1^2 f(x)\, dx = -2$. Use integration by parts to find $\int_1^2 xf'(x)\, dx$ **Solution:** 4

42. $\int_0^{\pi/4} \tan^3 x\ dx = (1 - \ln 2)/2$

43. Find $\int_0^{\pi/4} \tan^2 x\ \sec^4 x\ dx\ \ = 8/15$

44. Find $\int_0^{\pi} \sin^2 x\ dx = \pi/2$

45. Find $\int_0^{\pi} \cos^2 x\ dx = \pi/2$

46. Find $\int_0^{\pi} \sin^4 x\ \cos^2 x\ dx = \pi/16$

47. Find $\int_0^{\pi/3} \sec^2 \theta\ d\theta = \sqrt{3}$

48. Let $f(x) = x^2 + 1$. Find the point c in $[1, 7]$ such that $f(c)$ is the average value of f on $[1, 7]$.
    Solution: $c = \sqrt{19}$

49. Find $\int_1^2 \log_2(x)\ dx$**Solution:** $2 - 1/\ln 2$
    Solution: Use change of base $\log_a x = \frac{\ln x}{\ln a}$.

50. Find $\int_0^{\pi/2} x \sin x\ dx = 1$

51. Find $\int \sin^3 x\ dx = \frac{1}{3} \cos^3 x - \cos x + C$.