Math 132 - November 28, 2016

> **Section 11.9: Power Series as Functions**
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> | • Section 11.8: Radius and Interval of Convergence | • Operations on Power Series: Substitution, Addition, Multiplication, Division (difficult), Differentiation, Integration |

Warm-up Problems

1. **Clicker** Given the sum below:

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + \cdots = \frac{1}{1-x}$$

What is the sum:

$$1 - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 + \cdots$$

(a) $\dfrac{1}{1-x}$ (b) $\dfrac{1}{1+x}$ (c) $\dfrac{1}{1-x^2}$ (d) $\dfrac{1}{1+x^2}$ (e) Diverges

Class Problems

For the problems below, work hard to get the correct representation of the series. To do this, write out the series term-by-term and then try to find the pattern.

2. (Method: Substitution)
   Find a series for $\frac{1}{1+x^2}$

3. **Clicker** Find a power series for $f(x) = \dfrac{x^3}{4+x}$

   (a) $\displaystyle\sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n x^n$

   (b) $\displaystyle\sum_{n=0}^{\infty} 4^n x^n$

   (c) $\displaystyle\sum_{n=0}^{\infty} (-4)^{n+1} x^{n+3}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^{n+3}$

   (e) There is no series for $f(x)$.

4. Reindex the solution from the previous question so that the power of x is x^n.

5. (Method: Differentiation)
   Find a series for $\frac{1}{(1-x)^3}$

6. (Method: Integration. Note $\int \frac{1}{1+x^2}\, dx = \arctan x + C$)
   Find a series for $\frac{1}{1+x^2}$

7. Use your previous series to find a series for $\arctan(1) = \pi/4$

8. Find a series for $\frac{1}{1+x^{10}}$ and use your series to approximate $\displaystyle\int_0^{0.25} \frac{1}{1+x^{10}}\, dx$

9. (Method: multiplication)
   Use multiplication of power series to find a series for $\frac{1}{1-x} \cdot \frac{1}{1-x}$.
   (This is definitely a pain, but just distribute the terms out.)

   $$(1 + x + x^2 + x^3 + x^4 + \cdots)(1 + x + x^2 + x^3 + x^4 + \cdots) =$$

10. (Method: Long Division–Challenging and worth avoiding whenever possible)
    Use long division of power series to find a series for $\frac{\ln(1+x)}{\ln(1-x)}$
    Note: $\ln(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \cdots$
    and: $\ln(1 - x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \cdots$

11. Find power series for the following functions (you figure out the method(s) to use!)

    (a) $\dfrac{1}{4 + 3x} =$

    (b) $\dfrac{x}{(1 + x)^2} =$

    (c) $\dfrac{1 + 2x}{1 - x} =$