Math 132 - November 14, 2016
Solutions

Topics for Exam 3	
11.1: Sequences	11.4: Comparison (and Limit)
11.2: Geometric, Telescoping	11.5: Alternating Series
11.3: Integral Test	11.6: Absolute Convergence

1. Find the limit
   (a) $\lim_{n\to\infty} \frac{4n-3}{3n-4} = 4/3$
   (b) $\lim_{n\to\infty} \left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right) = \frac{1}{2}$
   (c) $\lim_{n\to\infty} \frac{1}{(1/10)^n} = \text{Diverges}$
   (d) $\lim_{n\to\infty} (\cos(1/n) + n\sin(1/n)) = 2$
   (e) $\lim_{n\to\infty} n^2 e^{-5n} = 0$
   (f) $\lim_{n\to\infty} \left(1+\frac{4}{n}\right)^n = e^4$
2. Find the sum, if possible
   (a) $\frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{243} + \cdots = 2/3$
   (b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n+1} - \frac{3}{n+2}\right) = 3/2$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^{n-1}} = 16$
   (d) $3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \frac{48}{625} + \cdots = 5$
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{8}{3^n} = 4$
   (f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{2^n} - \frac{1}{4^{n-1}}\right) = -1/3$
   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n} = \frac{1}{4}$
   (h) $\displaystyle\sum_{n=2}^{\infty} 3\left(\frac{-1}{2}\right)^n = 1/2$
   (i) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{(n+1)^2} - \frac{1}{n^2}\right) = -1$
   (j) $\displaystyle\sum_{n=1}^{\infty} (\cos(1/n) - \cos(1/(n+1))) = -1 + \cos 1$
3. Converge or diverge?
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}}$ **Solution:** Diverge
   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ **Solution:** Converge
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{1+n^3}}$ **Solution:** Diverge
   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ **Solution:** Diverge
   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n\sqrt{n}}$ **Solution:** Converge

(f) $\sum_{n=1}^{\infty} \frac{2^n}{n3^n}$ **Solution:** Converge

(g) $\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ **Solution:** Diverge

(h) $\sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}}$ **Solution:** Diverge

(i) $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ **Solution:** Diverge

(j) $\sum_{n=1}^{\infty} \frac{1}{n^\pi}$ **Solution:** Converge

(k) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^4+n+2}}$ **Solution:** Diverges

(l) $\sum_{n=1}^{\infty} \sqrt[n]{15}$ **Solution:** Diverges

(m) $\sum_{n=1}^{\infty} \frac{1}{5^n-1}$ **Solution:** Converges

(n) $\sum_{n=1}^{\infty} \frac{n^2}{n^3-1}$ **Solution:** Diverges

(o) $\sum_{n=1}^{\infty} \frac{1}{n!}$ **Solution:** Converges

(p) $\sum_{n=1}^{\infty} \frac{n+2}{n^{3/2}}$ **Solution:** Diverges

(q) $\sum_{n=1}^{\infty} \frac{3n^3}{n^4-2n^2}$ **Solution:** Diverges

(r) $\sum_{n=1}^{\infty} \frac{\sin n}{\sqrt{n^3+5}}$ **Solution:** Converges

4. Absolutely convergent, conditionally convergent, divergent?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+5}$ **Solution:** Absolutely Conv

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sin^2 n}{n^2}$ **Solution:** Absolutely Conv

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3+1}{n^4+1}$ **Solution:** Conditionally Conv

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n+1}$ **Solution:** Diverge

(e) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\ln n}{n^3}$ **Solution:** Absolutely Conv

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+2}$ **Solution:** Conditional Conv

(g) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^4+9}$ **Solution:** Absolutely Conv

(h) $\sum_{n=1}^{\infty} (-1)^n \cos(\pi/n)$ **Solution:** Diverges

5. $\sum_{n=1}^{\infty} \left(\frac{B}{B+2}\right)^n = 3$ Find $B = 6$

6. For $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$, find $S_{30} = 30/31$

7. $\sum_{n=2}^{\infty} (1+c)^{-n} = 2$. Find $c = (-1+\sqrt{3})/2$

8. $\sum_{n=1}^{\infty} \frac{(-1)^n a^{n+1}}{3^n}$. Find all a so that the series converges. **Solution:** $-3 < a < 3$