Math 132 - October 31, 2016

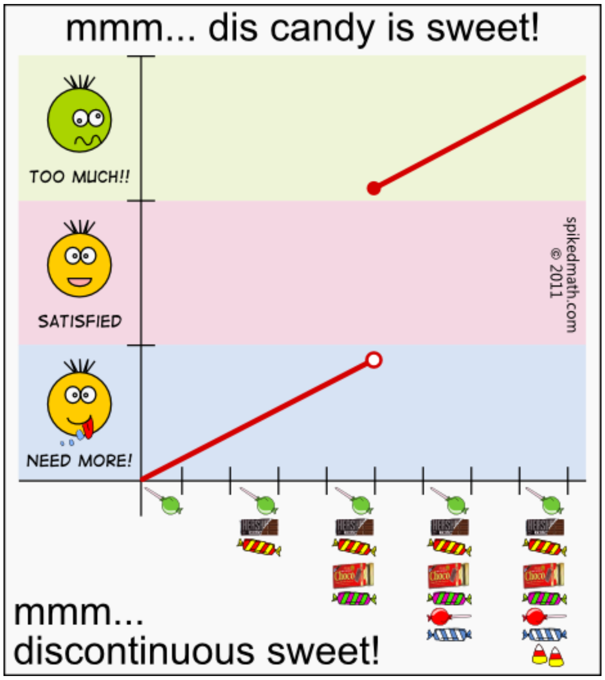
Solutions

Ideas from Section 11.2: Series

- An infinite sum is definited as the limits of the partial sums:

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty}\left(\sum_{i=1}^{n} a_i\right)$$

- Geometric Series 1: $s_n = \sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}$
- Geometric Series 2: $\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$
- Telescoping Series
- Harmonic Series: $\sum_{n=1}^{\infty} \frac{1}{n} = \infty$
- Test for Divergence:
If $\lim a_n \neq 0$ then $\sum_{n=0}^{\infty} a_n$ diverges



Warm-up Problems

1. Use long division to simplify

$$\frac{1-r^n}{1-r} = 1 + r + r^2 + \cdots + r^{n-1}$$

   (a) We can't conclude anything about this question until after WU completes its investigation of this instructor and this course. But that investigation won't be complete until January, way after the class is over. Thus, I have no way to answer this or any other question in this class.

   (b) r^{n-1}

   (c) $1 + r^{n-1}$

   (d) $1 - r^{n-1}$

   (e) $1 + r + r^2 + r^3 + \cdots + r^{n-1}$ $\boxed{\textbf{Correct}}$

Class Problems

2. Use Geometric series to compute the infinite sum:

   (a) $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n = \frac{(2/3)}{1-(2/3)} = 2$

   (b) $\sum_{n=2}^{\infty}\left(\frac{2}{3}\right)^n = \frac{4}{3}$

   (c) $\sum_{n=2}^{\infty}\left(\frac{2}{3}\right)^{n-2} = 3$

   (d) $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n = \frac{1}{1+2/3} = \frac{3}{5}$

(e) $\displaystyle\sum_{n=2}^{\infty} \frac{2^n \cdot 3^{n-3}}{5^{2n+3}} = \sum_{n=2}^{\infty} \left(\frac{3^{-3}}{5^3}\right)\left(\frac{2\cdot 3}{25}\right)^n = \left(\frac{1}{9\cdot 125}\right)\left(\frac{6}{25}\right)^2\left(\frac{1}{1-6/25}\right)$

3. Use Geometric series to compute the partial sums

(a) $\displaystyle\sum_{n=1}^{4} \left(\frac{2}{3}\right)^n = \frac{(2/3)\cdot(1-(2/3)^4)}{1-(2/3)}$

(b) $\displaystyle\sum_{n=1}^{4} \left(\frac{2}{3}\right)^{n-1} = \frac{(1)\cdot(1-(2/3)^4)}{1-(2/3)}$

(c) $\displaystyle\sum_{n=4}^{7} \left(\frac{2}{3}\right)^n = \frac{(2/3)^4\cdot(1-(2/3)^4)}{1-(2/3)}$

4. Use telescoping series to compute the series.

(a) $\boxed{\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2-n} = \sum_{n=1}^{\infty}\left(\frac{1}{n-1}-\frac{1}{n}\right)}$

$s_N = 1 - \dfrac{1}{N}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2-n} = 1$

(b) $\boxed{\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3} = \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{n+1}-\frac{1}{n+3}\right)}$

$s_N = \dfrac{1}{2}\left(\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{n+2}-\dfrac{1}{n+3}\right)$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3} = \frac{5}{12}$

(c) $\boxed{\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{k+1}{k}\right) = \sum_{n=1}^{\infty}(\ln(k+1)-\ln k)}$

$s_N = \ln n - \ln 1$

$\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{k+1}{k}\right) = \text{Diverges}$